Types of numbers

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1) **Natural Number** :- (नैसर्गिक संख्या) :- A number started from 1, 2, 3, 4 .... ∞ .

2) **Whole Number** :- (W) :- ~~A number~~ natural number with zero is called as Whole number. e.g. 0, 10, 20, ... ∞

3) **Integer** (Z) :- Zero natural number and negative natural number - $-2, -1, 0, 1, 2, 3, \ldots \infty$

4) **Rational number** :- (Q) :- A number of type p/q , $p, q \in Z$ & $q \neq 0$ called as rational number.

5) **Irrational number** (Q^c) :- e.g $\sqrt{5}, \sqrt{7}, \sqrt{3} \ldots \infty$

6) **Real number** (R) :- (1-set) Rational number + Irrational number.

Qudratic equation :-

An equation in which the highest degree is '2' is called as qudratic equation e.g $x^2+6x+6x+5=0$

Standard form of qudratic equations -

$ax^2+bx+c=0$ a, b, & c (Real number) which is $a \neq 0$

Reason :- If we take $a=0$, then above equation becomes, $0x^2+bx+c=0$.

$\Rightarrow 0+bx+c=0$

$bx+c=0$ this is not a quadratic equation.

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Mathematical Symbols & its meaning :-

1) $\in$:- belongs to
2) $\notin$:- does not belongs to.
3) $\exists$:- there exist
4) $\nexists$:- there does not exist
5) $\forall$:- for all
6) $\because$:- so that
7) $\therefore$:- reason.

Multiplication :-

$(+) \times (+) = (+)$

$(+) \times (-) = (-)$

$(-) \times (+) = (-)$

$(-) \times (-) = (+)$

above rule same for division.

e.g. Solve the following quadratic equation.

$$x^2+5x+6=0$$

$$M_1 = x^2+2x+3x+6=0$$

$$x(x+2)+3(x+2)=0$$

$$(x+2)(x+3)=0$$

$$x+2=0 \text{ or } x+3=0$$

$$\boxed{x=-2} \text{ or } \boxed{x=-3}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

formula method

$$x^2+5x+6=0$$

comparing with $ax^2+bx+c=0$

$$x^2+5x+6=0$$

$$x^2+2x+3x+6=0$$

$$x^2(x+2)+3(x+2)=0$$

$$(x+2)(x+3)=0$$

$$x+2=0 \text{ or } x+3=0$$

$$x=-2 \quad x=-3$$

formula method

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2-4\times1\times6}}{2\times1}$$

$$x = \frac{-5 \pm \sqrt{25-24}}{2}$$

$$x = \frac{-5+1}{2} \text{ or } \frac{-5-1}{2}$$

$$x = \frac{-4}{2} \text{ or } \frac{-6}{2}$$

$$\boxed{x=-2} \quad \boxed{x=-3}$$

1. Logarithm

Logarithms:-

Logarithm is the power to which a number must be raised in order to get some other numbers.

It is discovered by scottish Laird, John Napier Marchiston.

He introduce logarithm as way to simplify the calculations.

~~He introduce logarithm as way to simplify~~ the

Logarithms is a shortes method to find the value of 'x' if we have an equation of type $\boxed{y=a^x}$ {y is equal to a to the power of x}

$x=\log_a y$:- [it is read as x is equal to log of y to base a]

equation y is in exponential form

$y=a^x$ - exponential form.

$\Updownarrow$

$x=\log_a y$ = logarithm form.

$y>0$, $a>0$ & $a\neq 1$

Types of logarithm:-

1) Common Logarithm :- Logarithm of the base '1' are called as common logarithm eg $\log_{10} 358$

2) Natural Logarithms:- Logarithm of the base 'e' called as natural Logarithm e.g $\log_e 358$.

e :- 2.718

$$\boxed{\log_e b = 2.303 \log_{10} b}$$

Basic properties of logarithm:-

1) $\log_a 1 = 0$

Logarithm of '1' with any base is always be zero.

Proof :- given that,

$\log_a 1 = 0$

Consider $\log_a 1 = x$ —— ①

Convert it into exponential form we get, $1 = a^x$ $a^x = 1$ — (ii)

but we know that, $a^0 = 1$

Show that equation becomes

$a^x = a^0$ $\boxed{x=0}$

② $\log_a a = 1$; $a > 0$, $a \neq 1$

Logarithm of $= a$ with any base is always be one.

Proof :- given that

$\log_a a = 1$ consider $\log_a a = x$ —— ①

Convert its exponential form is $a = a^x$ — (ii)

but we know that, $a^1 = a$

$\Rightarrow a^1 = a^x$ $\Rightarrow$ $x = 1$

put this in equation ①

$\boxed{\log_a a = 1}$ Hence proved.

Rule of logarithms:-

Product Rule

① $\log_a (m.n) = \log_a m + \log_a n$; $a > 0$, $a \neq 0$

$\log_a m + \log_a n = \log_a (m.n)$

Proof :- given that $\log (m.n) = \log_a m + \log_a n$

Consider, $\log_a (m) = p \Rightarrow m = a^p$ —— (i)

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$\log_a(n) = q \Rightarrow n = a^q$ — (ii)

$m.n. = a^p . a^q$

$m.n = a^{p+q}$

$p+q = \log_a(m.n)$

by equation (i) & (ii)

$L.H.S = R.H.S$

$$\boxed{\log_a(m) + \log_a(n) = \log_a(m.n)}$$

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▽ **Quotient Rule :-**

$\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n), \ a>0 = a \neq 1$

or

$\log_a(m) = \log_a(n) = \log_a\left(\frac{m}{n}\right) \ a>0 = a\neq 1$

▽ **Power Rule :-**

$\log_a m^n = n \log_a m : a>0, a\neq 1$

Solve the following

1) $\log\left(\frac{2}{3}\right) + \log\left(\frac{4}{5}\right) - \log\left(\frac{8}{15}\right)$

$= \log\left(\frac{2}{3}\times\frac{4}{5}\right) = \log\left(\frac{8}{15}\right)$

$\log\frac{8}{15} = \log\frac{8}{15}$

$\log\left(\frac{\cancel{8/15}}{\cancel{8/15}}\right)$

$\log = (1) = \boxed{\log = 0}$

2) $\log\left(\frac{9}{10}\right) - \log\left(\frac{3}{5}\right) + \log\left(\frac{3}{2}\right) = ?$

$\log\left(\frac{\cancel{9}^{3}/\cancel{10}^{2}}{\cancel{3}^{1}/\cancel{5}^{1}}\right) = \log\left(\frac{3}{2}\right)$

$\log\left(\frac{3}{2}\times\frac{3}{2}\right)$

$$\boxed{\log\left(\frac{9}{4}\right)}$$

Change of base Rule :-

$$\boxed{\log_a m = \frac{\log_n m}{\log_n a}, \; n>0,\; n\neq 1}$$

e.g -① $\log_5 \frac{9}{10}$ change The base into 7.

$$\boxed{\log 5 = \frac{\log_{a} 7}{\log 7} = \frac{9/10}{5}}$$

② $\log_9 (2)$ change the base into '4'

$$\boxed{\log 9 = \frac{\log_4}{\log 4} = \frac{2}{9}}$$

Examples on ~~logari~~ Logarithm

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Ex.1 convert exponetial form to logarithmic From.

i) $6^0 = 1$

$1 = 6^0$ ...[This is exponential from. $y = a^x$]

Here, $y = 1, a = 6, x = 0$

$\therefore$ Now, logarithmic form is, $x = \log_a y$

$\boxed{0 = \log_6 1}$ - we can say $\boxed{\log_6(1) = 0}$

ii) $2^3 = 8$

$8 = 2^3$ ..[This is exponential form. $y = a^x$]

Here $y = 8, a = 2, x = 3$

$\therefore$ Now, logarithmic form is. $x = \log_a y$

$\boxed{3 = \log_2 8}$ - we can say $\boxed{\log_2 8 = 3}$.

Ex.2. Convert into exponential form.

i) $\log_8 8 = 1$

$1 = \log_8 8$ ... [This is logarithmic form, $x = \log_a y$]

Here, $x = 1$, $a = 8$, ~~x=8~~ $y = 8$

Now, exponential form is,

$y = a^x$

$\boxed{8 = 8^1}$

ii) $\log_4 \frac{1}{16} = -2$

$-2 = \log_4 \frac{1}{16}$ .. [This is logarithmic form, $x = \log_a y$]

Here, $x = -2$, $a = 4$, $y = \frac{1}{16}$

Now, exponential form is

$y = a^x$

$\boxed{\frac{1}{16} = 4^{-2}}$

E.x. Find the value of the following.

i) $\log_5 625$

= let, $x = \log_5 625$

... $x = \log_a y$ (This is logarithmic)

Here, $y = 625$, $a = 5$, $x = x$

∴ Exponential form is

$y = a^x$

$625 = 5^x$

$[5]^4 = 5^x$

$x = 4$

Equation (1) becomes

$4 = \log_5 625$

∴ We can say that

$\boxed{\log_5 625 = 4}$

ii) $\log_3 81$

Let, $x = \log_3 81$

.... $x = \log_a y$ (This is logarithmic)

Here, $y = 81$, $a = 3$, $x = x$

∴ Exponential form is

$y = a^x$

$81 = 3^x$

~~$[3]^4 = [3]^x$~~ $3^4 = 3^x$

$x = 4$

Equation (1) becomes

$4 = \log_3 81$

∴ We can say that

$\boxed{\log_3 81 = 4}$

iii) $\log_{16} 4$

let, $x = \log_{16} 4$

$\therefore x = \log_a y$ [This is logarithmic form] -(I)

Here, $y = 4$, $a = 16$, $x = x$.

Exponential form is,

$y = a^x$

$4 = 16^x$

~~$4 = [16^2]^x$~~

$4 = [4^2]^x$

$4[2x]$

$2x = 1$

$x = \frac{1}{2}$

Equation (I) becomes,

$\frac{1}{2} = \log_{16} 4$

$\boxed{\log_{16} 4 = \frac{1}{2}}$

iv) $\log_{16} 64$

let, $x = \log_{16} 64$

$\therefore x = \log_a y$ [This is logarithmic form]

Here, $y = 64$, $a = 16$, $x = x$

Exponential form is

$y = a^x$

$64 = 16^x$

$(4)^3 = (4^2)^x = 4^{2x}$

$3 = 2x$

$x = \frac{3}{2}$

Equation (I) becomes,

$\frac{3}{2} = \log_{16} 64$

we can say that

$\boxed{\log_{16} 64 = \frac{3}{2}}$

v) $\log_{2\sqrt{3}} 12$

let, $x = \log_{2\sqrt{3}} 12$

$\therefore x = \log_a y$ [This is logarithmic form] - I

~~Her~~

Here $y = 12$ $a = 2\sqrt{3}$ $x = x$

Exponential form is $y = a^x$.

$12 = [2\sqrt{3}]^x$

$(2\sqrt{3})^2 = [2\sqrt{3}]^x$

$x = 2$

Equation (I) become, $2 = \log_{2\sqrt{3}} 12$

we can say that,

$\boxed{\log_{2\sqrt{3}} 12 = 2}$

vii) $\log_9 (\log_2 2)$

let, $x = \log_9 [\log_2 2]$

But as we know,

$\log_2 2 = 1$

$\therefore$ Equation (I) becomes,

$x = \log_9 (1)$

$\boxed{x = 0}$

v) $\log_3 \frac{1}{9}$

let, $x = \log_3 \frac{1}{9}$ $\therefore x = \log_a y$ [This is logarithmic fo

Here $y = \frac{1}{9}$, $a = 3$, $x = x$

Exponential form is $y = a^x$

$\frac{1}{9} = 3^x$

$\frac{1}{3^2} = 3^x$

$3^{-2} = 3^x$

$x = -2$

Equation (I) beomes

$-2 = \log_3 \frac{1}{9}$ we can say that,

$\boxed{\log_3 \frac{1}{9} = -2}$

Ex. 4 Solve the following:

i) $\log 3 + \log 5 - \log 4$

$\log [3 \times 5] - \log 4$

$\log 15 - \log 4$

$\boxed{\log \frac{15}{4}} = \log m - \log n = \log \left[\frac{m}{n}\right]$

iii) $\frac{1}{2} \log 16 + \frac{1}{3} \log 27$

$\log 16^{\frac{1}{2}} + \log 27^{\frac{1}{3}}$

$\left[\log 4^{2 \cdot \frac{1}{2}}\right] + \left[\log 3^{3 \cdot \frac{1}{3}}\right]$

$\log 4 + \log 3$

$\log [4 \times 3] = \boxed{\log [12]}$

iv $3 \log_{10} 2 - \frac{1}{2} \log_{10} 36 + 1$

$\log_{10} 2^3 - \log_{10} 36^{\frac{1}{2}}$

By using $\log m^n = n \log m$ and $\log_a a = 1$

$\log_{10} 8 - \log_{10} 6^{2 \times \frac{1}{2}} + \log_{10} 10$

$\cdots 2^3 = 8$ and $6^2 = 36$

$= \log_{10} \left[\frac{8}{6}\right] + \log_{10} 10 \cdots \log m - \log n = \log \left[\frac{m}{n}\right]$

$= \log_{10} \left[\frac{8}{6}\right] + \log_{10} 10$

$= \log_{10} \left[\frac{8}{6} \times 10\right] \cdots \log m + \log n = \log [m n]$

$= \left[\frac{40}{3}\right] = \boxed{\log_{10} \left[\frac{40}{3}\right]}$

v) $\log (x^2 - 3x + 2) - \log (x-1) + \log (x-2)$

$= \log (x^2 - 3x + 2) + \log (x-2) - \log (x-1)$

$= \log [(x^2 - 3x + 2)(x-2)] - \log (x-1)$

$\cdots \log m + \log n = \log (m.n)$

$= \log [(x-2)(x-1)(x-2)] - \log (x-1)$

$= \log \left[\frac{[x-2]\cancel{[x-1]}(x-2)}{\cancel{[x-1]}}\right]$

$\cdots \log m - \log n = \log \left[\frac{m}{n}\right]$

$= \log [(x-2)^2]$

$\boxed{2 \log (x-2)} \cdots \log m^n = n \log m$

find the value of $x = ?$ - Ex. 5

i) $\log_3 x - \log_3 (x-2) = 3$

$\log_3 x - \log_3 (x-2) = 3$

$\log_3 \left[\frac{x}{x-2}\right] = 3$

$\cdots \log_a m - \log_a n = \log_a \left[\frac{m}{n}\right]$

$3 = \log_3 \left[\frac{x}{x-2}\right] \cdots x = \log_a y$

1. This is logarithmic form,
2. Here, $x = 3$, $a = 3$, $y = \frac{x}{x-2}$

Let us convert it into exponential form,

$y = a^x$

$\frac{x}{x-2} = 3^3$

$\frac{x}{x-2} = 27$

$x = 27 [x-2]$

$x = 27x - 54$

$x - 27x = -54$

$-26x = -54$

$\boxed{x = \frac{54}{26} = \frac{27}{13}}$

vi) $\log_e(x+1) + \log_e x = \log_e 6$

$\log_e[(x+1)(x)] = \log_e 6$

$(x+1)(x) = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x+3 = 0$

or $x-2 = 0$

$x = -3$ or $x = 2$

$(x+1)(x) = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$(x+3) = 0$ or $x-2 = 0$

$x = -3$ or $x = 2$

Putting $x = -3$ in

$\log_e(x+1) + \log_e x = \log_e 6$

$\log_e(-3+1) + \log_e -3 = \log_e 6$

$\log_e(-2) + \log_e(-3) = \log_e 6$

Which is not satisfied as logarithm of negative term does not exist

$\therefore$ $x = -3$ is not applicable.

Now, Putting $x = 2$ in

$\log_e(x+1) + \log_e x = \log_e 6$

$\log_e(2+1) + \log_e 2 = \log_e 6$

$\log_e(3) + \log(2) = \log_e 6$

$\log_e[3(2)] = \log_e 6$

$\log_e 6 = \log_e 6$

i.e. Equation is satisfied

we can take $\boxed{x = 2}$

vii) $\log_3(x+6) = 2$

$2 = \log_3(x+6)$

... [This is logarithmic form, $x = \log_a y$]

Here, $a = 3$, $x = 2$, $y = (x+6)$

Let us convert it in exponential form we get,

$y = a^x$

$x + 6 = 3^2$

$x + 6 = 9$

$x = 9 - 6$

$\boxed{x = 3}$

Exercise

1. Write the following results into their equivalent logarithmic form.

a) $(125)^{\frac{1}{3}} = 5$

given $(125)^{\frac{1}{3}} = 5$

$(125)^{\frac{1}{3}} = 5$ this is exponention form its convert into to logarithmic form.

Here, $a = 125, x = \frac{1}{3}, y = 5$

$\cancel{\log_{125}\frac{1}{3} = 5}$ $\boxed{\log_{125} 5 = \frac{1}{3}}$

b) $(243)^{\frac{1}{5}} = 3$

given $(243)^{\frac{1}{5}} = 3$

$(243)^{\frac{1}{5}} = 3$ this is exponention form its convert into logarithmic form

Here, $y = 3, x = \frac{1}{5}, a = (243)$

$\cancel{\log_{243}\frac{1}{5} = 3}$ ... $\log_a y = x$

$\boxed{\log_{243} 3 = \frac{1}{5}}$

c) $(8)^3 = 512$

$(8)^3 = 512$ this is exponention form and it's convert in to logarithmic form $\log_a y = x$.

Here, $y = 512, x = 3, a = 8$

$\boxed{\log_8 512 = 3}$

d) $(9)^4 = 6561$

$(9)^4 = 6561$ this is exponention form and its convert into logarithmic form $\log_a y = x$

Here, $y = 6561, x = 4, a = 9$

$\boxed{\log_9 6561 = 4}$

2. Write the following results into their equivalent Exponential form.

$\log_9 6561 = 4$

$\log_9 6561 = 4$ is convert into Exponential form. ... $a^x = y$

Here, $y = 6561, x = 4, a = 9$

$\boxed{(9)^4 = 6561}$

4. Express the following as a single logarithmic using properties (laws) of logarithm

$5\log_7 5 + 3\log_7 3 - 9\log_7 9$

$\log_7 5^5 + \log_7 3^3 - \log_7 9^9$ ... $\log m^n = n\log m$

$\log(5^5 + 3^3) - \log_7 9^9$

$\boxed{\log\left[\frac{5^5 + 3^3}{9^9}\right]}$

$\log 7 = \frac{5^5 + 3^3}{9^9}$

3. Determine the value of following

$2\log_7 5 - 3\log_7 3 + 4\log_7 3 = \log_7 x$

$\log_7$ take commone

$\log_7 = \frac{5^2 - 3^3}{3^4} = \log_7 x$

$\log_7 = \frac{25-9}{12} = \log_7 x$

$\log_7 = \frac{16}{12} = \log_7 x$

$\log_7 = \frac{\cancel{16}^{8}}{\cancel{12}_{6}} = \log_7 x$

$\log_7 = \frac{\cancel{8}^{4}}{\cancel{6}_{3}} = \log_7 x$

$\log_7 = \frac{4}{3} = \log_7 x$

$x = \frac{4}{3}$

$\log_7 x = \boxed{\log_7 \frac{4}{3}}$

$\log_7 5^2 - \log_7 3^3 + \log_7 3^4 =$

$\log_7 \left[\frac{5^2}{3^3}\right] + \log_7 3^4$

$\log_7 \left[\frac{5^2}{3^3} \times 3^4\right]$

$\log_7 \left[\frac{25}{27} \times 81\right]$

$\boxed{\log_7 x = 75}$

E.x.6 Prove that, $\log(1+2+3) = \log 1 + \log 2 + \log 3$.

L.H.S $\log(1+2+3)$

$= \log 6$

$= \log(1\times 2\times 3)$ --- $6 = 1\times 2\times 3$

$= \log(1) \times \log(2) \times \log(3)$ - [$\log(m\times n)$ $\log m + \log n$)

$=$ R.H.S

Ex.7 Show that, $m^{\log n} = n^{\log m}$

Let, $m^{\log n} = x$.

Taking log on both side

$\log m^{\log n} = \log x$.

$\log n \cdot \log m = \log x$ -- ①

... $\log m^n = n \log m$

Let $n^{\log m} = y$

Taking log on both sides,

$\log n^{\log m} = \log x = \log y$.

$\log m \cdot \log n = \log y$ -- ②

-- $\log m^n = n\log m$

From equation (1) and (2) we get,

$\log n \cdot \log m = \log x = \log y$

$\log x = \log y$

$x = y$ -- by equality of logarithm

$m^{\log n} = n^{\log m}$

... $x = m^{\log n}$ and $y = n^{\log m}$.

Ex 8. Solve the equation, $\frac{\log(x+6)}{\log x} = 2$

Given: $\frac{\log(x+6)}{\log x} = 2$

$\log(x+6) = 2\log x$

$\log(x+6) = \log x^2$

By equality rule of logarithms

$\log m = \log n$

$m = n$

$x + 6 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x = 3}$ or $\boxed{x = -2}$

Now, $\log x = \log 3$ is allowed but $\log x = \log -2$ is not allowed because logarithm of negative term is not applicable.

$\boxed{x = 3}$

Ex9. Solve, to find the value of $x = ?$

$$\log(2+x) - \log(2-x) = 1$$

$$\log_e \left[\frac{2+x}{2-x}\right] = 1$$

Base is not given means base is e.

- $\log m - \log n = \log\left[\frac{m}{n}\right]$

$$1 = \log_e \left[\frac{2+x}{2-x}\right]$$

$x = \log_a y$ {This is Logarithmic form}

When $x = 1$, $a = e$, $y = \frac{2+x}{2-x}$

Convert the equation in exponential form.

~~$y = a+x$~~ $y = a^x$

$$\frac{2+x}{2-x} = e^1$$

$$2+x = e(2-x)$$

$$2+x = 2e - ex$$

$$ex + x = 2e - 2$$

$$x(e \cancel{+} 1) = 2(e-1)$$

$$\boxed{x = 2\left[\frac{e+1}{e-1}\right]}$$

Ex.10 Solve, $\log_3 (x+1) - \log_3 (x-4) = 2$

- given, $\log_3 (x+1) - \log_3 (x-4) = 2$.

$\log_3 \left[\frac{(x+1)}{(x-4)}\right] = 2$ -- This is Logarithmic form.

- $\log_a m - \log_a n = \log_a \left[\frac{m}{n}\right]$

Let, $2 = \log_3 \left[\frac{(x+1)}{(x-4)}\right]$

$x = \log_a y$

Here $x = 2$, $a = 3$, $y = \frac{x+1}{x-4}$

Convert into Exponential Form.

$$y = a^x$$

$$\frac{x+1}{x-4} = (3)^2$$

$$\frac{x+1}{x-4} = 9$$

$$x+1 = 9(x-4)$$

$$x+1 = 9x - 36$$

$$x - 9x = -36 - 1$$

$$8x = 37$$

$$\boxed{x = \frac{37}{8}}$$

Ex 11. $\log_4 x + \log_4(x+6) = \log_4(5x+12)$

Given: $\log_4 x + \log_4(x+6) = \log_4(5x+12)$

$\log_4[(x)(x+6)] = \log_4(5x+12)$

$\because \log_a m + \log_a n = \log_a(m \cdot n)$ —(1)

By ~~st~~ using equality property of logarithum,

$\log_a m = \log_a n \quad \therefore m = n$

Equation (1) becomes,

$x(x+6) = 5x+12$

$\therefore x^2+6x-5x-12=0$

$x^2-x-12=0$

$(x+4)\ (x-3)=0$

$x+4=0$ or $x-3=0$

$x=-4$ or $x=3$

$\therefore \boxed{x=-4 \text{ or } x=-3}$

Ex 12 prove that,

$\log_{10}(\sqrt{x^2+1}-x) + \log_{10}(\sqrt{x^2+1}+x) = 0$

L.H.S

$\log_{10}(\sqrt{x^2+1}-x) + \log_{10}(\sqrt{x^2+1}+x)$

$\log_{10}(\sqrt{x^2+1}-x) \cdot \log_{10}(\sqrt{x^2+1}+x)$

$\because \left[\log_{10} m + \log_{10} n = \log_{10} \frac{m}{n}\right]$

$\log_{10}[\sqrt{[x^2+1]^2}-x^2]$

~~$\log_{10}[\sqrt{}$~~ $\log_{10}[x^2+1-x^2]$

$\log_{10}(x^2+1-x^2)$

$\boxed{\log_{10}(1)=0}$ = R.H.S

Ex. 13 prove that, $\log(\log x^4) - \log(\log x^6) = \log\left(\frac{2}{3}\right)$

~~L.H.S $\log\left(\frac{\log x}{\log 4}\right)$~~

L.H.S $\log\left[\frac{\log x^4}{\log x^6}\right] = \log\left(\frac{2}{3}\right)$ ... $\log m - \log n = \log\left[\frac{m}{n}\right]$

$\log\left[\frac{4\log x}{6\log x}\right]$

$\log\left[\frac{4}{6}\right] = \log\left[\frac{2}{3}\right] = R.H.S.$

Ex 14. solve : $\log_3(x+5) - \log_3(2x-1) = 2$

$\log_3 \frac{[x+5]}{[2x-1]} = 2$

-- $\log_a m - \log_a n = \log_a\left[\frac{m}{n}\right]$

$2 = \log_3 \frac{[x+5]}{[2x-1]}$

$x = \log_a y$ (This is Logarithmic form)

Here, $x = 2, a = 3, y = \frac{(x+5)}{(2x-1)}$

Convert it into exponential form

$y = a^x$

$\frac{[x+5]}{[2x-1]} = (3)^2$

$\frac{(x+5)}{(2x-1)} = 9$

$x+5 = 9(2x-1)$

$x+5 = 18x - 9$

$18x - x = 5 + 9$

$17x = 14$

$$\boxed{x = \frac{14}{17}}$$

Ex 15. If $p^2+q^2 = 7pq$ then show that $\log\left(\frac{p+q}{3}\right) = \frac{1}{2}(\log p + \log q)$

Given $p^2+q^2 = 7pq$

Adding $2pq$ on both side.

$p^2+2pq+q^2 = 7pq+2pq$

$(p+q)^2 = 9pq$

.... $(p+q)^2 = p^2+2pq+q^2$

$\frac{(p+q)^2}{9} = pq$

$\left(\frac{p+q}{3}\right)^2 = pq$

Taking log on both side.

$\log\left(\frac{p+q}{3}\right)^2 = \log[pq]$

$2\log\left[\frac{p+q}{3}\right] = \log pq$ ... $\log m^n = n\log m$.

$\log\left[\frac{p+q}{3}\right] = \frac{1}{2}(\log p + \log q)$

Hence proved.

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Ex.16 If $\log\left(\frac{p-q}{5}\right) = \frac{1}{2}(\log p + \log q)$;
show that, $p^2+q^2=27pq$

Given $\left(\frac{p-q}{5}\right) = \frac{1}{2}(\log p + \log q)$

$\log\left(\frac{p-q}{5}\right) = \frac{1}{2}\log(pq)$ ... $\log m^n = n\log m$

$\log\left(\frac{p-q}{5}\right) = \log(pq)^{\frac{1}{2}}$

Now dropping logarithm from both sid
we get,

$$\frac{p-q}{5} = (pq)^{\frac{1}{2}}$$

Taking square on both side we get,

$$\left(\frac{p-q}{5}\right)^2 = \left[(pq)^{\frac{1}{2}}\right]^2$$

$$\left(\frac{p-q}{25}\right) = pq$$

$$p^2-2pq+q^2=25pq$$

$$p^2+q^2=25pq+2pq$$

$$p^2+q^2=27pq$$

$$\boxed{p^2+q^2=27pq}$$

Hance Prooved